EFFECTIVE INSTRU

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Teachers, administrators, parents, and researchers generally agree that teachers are the crucial factor in student achievement. The National Mathematics Advisory Panel (2008) has noted that differences in instruction lead to significant differences in student achievement, thus highlighting the importance of teachers in learning. Even though class size, student placement, and class offerings influence students' mathematical understanding, the quality of teacher instruction has been found to have a stronger effect (Wright, Horn, and Sanders 1997). Effective teachers focus on effective instruction.

What constitutes effective instruction? An answer to this question is more complex than it initially seems. No singular, concrete recommendation or outline of effective instruction exists. Effective instruction is multifaceted, dependent largely on the context and, consequently, on numerous variables. Although *effective instruction* is difficult to define, in my experience—and as the work of mathematics education specialists and researchers indicates—three key features of quality instruction stand out:

1. Teaching conceptually for understanding (Horowitz et al. 2005; NCTM 2009; National Mathematics Advisory Panel 2008; Shepard et al. 2005)
2. Making connections within content, including accessing prior knowledge (Brophy and Good 1986; NCTM 2009; Shell et al. 2009; Shepard et al. 2005; Wharton-McDonald et al. 1998)
3. Directing attention through student engagement (Brophy and Good 1986; Marzano, Pickering, and Pollock 2001; Shell et al. 2009; Wharton-McDonald et al. 1998)

As an algebra instructional coach, I have observed and participated in a variety of instructional interactions with high school algebra teachers and students. In the classrooms I have worked in, these three main instructional features are often the key ingredients to effective lessons; conversely, when instruction is not effective, these are the missing components. The following examples of effective instruction are drawn from my experience as a teacher and an instructional coach.

CONCEPTUAL UNDERSTANDING

The National Mathematics Advisory Panel (2008) came to the conclusion that curriculum should include conceptual understanding. In addition to focusing on procedures and computation, effective instruction facilitates deeper investigation into the "why?" of mathematics. By focusing

When it comes to student achievement, teachers and instruction matter most.

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instruction on the conceptual meaning of mathematics, a teacher confronts previously learned "memorize and repeat" behaviors while encouraging students' understanding and ability to think mathematically.

The following classroom example shows how one teacher taught conceptually for understanding.

> Ms. Sheer [all names in this article are pseudonyms] was frustrated by the repeated mistakes students made when solving and graphing inequalities. It seemed as if no matter how many times she said, "When you multiply or divide by a negative, you switch the inequality symbol," students still forgot. She decided to focus on the conceptual understanding behind solving inequalities and ask students to determine why the inequality symbol changed directions when multiplying or dividing by a negative number.
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> Ms. Sheer had her students begin with the true statement $-4 < 6$. They graphed the two numbers on a number line and discussed what would happen to the statement and the graph if they added a positive number to both sides of the inequality. What happened to the inequality symbol? What happened to the plotted points? Why? (See **fig. 1**.)
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> Continuing with the original statement, students added a negative number, subtracted a positive number, subtracted a negative number, and so on until every operation was performed with both a positive and a negative integer. Students discussed what was happening when they multiplied and divided by a number less than zero. What happened to the inequality symbol? What happened to the plotted points? Why did the inequality symbol change only with these two examples? (See **fig. 2**.)
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> Through scaffolded inquiry, students discovered what happens to an inequality symbol when an inequality is multiplied or divided by a negative number. Using mathematical reasoning, number sense, and visual representation, they now had the "why" behind the rule.

Ms. Sheer realized that for her students to consistently solve inequalities correctly, they

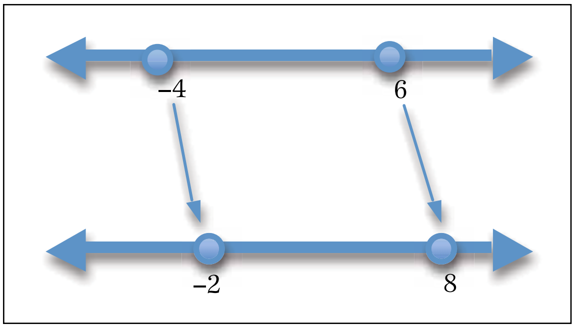

Fig. 1 Adding 2 to both sides of an inequality can be represented algebraically and graphically.

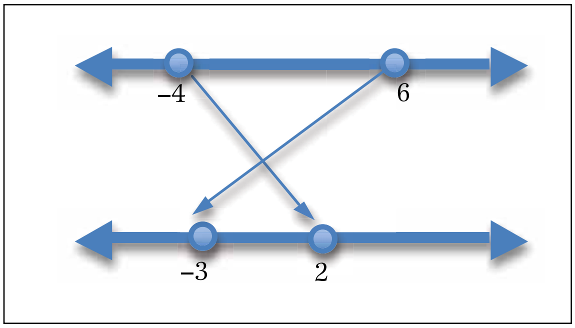

Fig. 2 When students multiply or divide a true inequality by a negative number (in this case, -2), the rule makes more sense to them.

needed to understand the big idea behind the rule. Although procedural knowledge would be important in completing the problems, students first needed to develop conceptual knowledge of inequalities. The focus on student conceptual understanding helps students learn new topics more easily (NCTM 2009) and retain the information; in addition, it increases the likelihood that students can transfer their knowledge to new learning situations (Shepard et al. 2005).

MATHEMATICAL CONNECTIONS AND PRIOR KNOWLEDGE

Making mathematical connections, a process that includes accessing prior knowledge, is another feature of effective instruction. Mathematics should be taught as an intricate network of concepts and skills that build on one another, rather than as a list of isolated procedures. According to NCTM (2009), the purpose of high school mathematics is to develop students who can reason and make sense of their conclusions. Creating mathematical connections is the underlying means toward that goal and is, therefore, an essential part of effective mathematics instruction. When a teacher emphasizes mathematical connections, students are more likely to understand and retain the information (NCTM 2009). Therefore, highlighting curricular connections is essential if we want to increase student understanding.

The more ways information is connected, the more ways it can be accessed. When a teacher helps students link new concepts to other mathematical understanding, they have a better chance of retrieving and using the new knowledge later because of the vast number of neurological connections (Shell et al. 2009). Teachers are essentially helping students retain what they learn by emphasizing curricular connections (NCTM 2009). Material taught in isolation, without connections to prior knowledge, is stored as a separate piece of information and can be retrieved only by a specific trigger, making the retention and retrieval of the new learning limited and more difficult. Therefore, the more connections a teacher can facilitate, the more easily students will be able to retrieve and use what they have learned.

The following example from Mr. Conrey's class demonstrates the power of curricular connections.

> Mr. Conrey believed that it was important for his ninth-grade students to understand rather than memorize slope-intercept form and how it connects with other mathematics. His students had been using tables to write function rules since the beginning of the year. They had gotten quite good at looking at the y-values for a pattern and then using that information to determine a rule using the ordered pairs. Today's lesson was going to build on the students' understanding of tables, function rules, and slope, which had been the focus in class the previous two days.
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> As a warm-up activity, students were given a table of values and asked to write the function rule, graph the ordered pairs, and then determine the slope from the graph. Mr. Conrey was confident in the students' abilities to complete these tasks. He challenged each student to find how the slope was related to the function rule and the table of values. He could almost see light bulbs come on above students' heads as they began to realize that the slope was the coefficient of x in the equation as well as the pattern in the y-values of the table. Rather than address this connection with the whole class at this point, Mr. Conrey had students write down their individual conjectures until they could further investigate the relationship.
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> Mr. Conrey then gave teams of students a new function rule. Each team created a table and a graph based on their assigned linear equation and found the slope. Teams tested their earlier conjectures by again finding how the slope related to the table of values and the equation. Students around the room were beginning to realize that the slope was the coefficient of x in the given equation and also the constant rate of change in their tables.

As a class, students made the connection that the "pattern in the table" they had previously been finding was in fact the slope. And the slope, which they had previously been able to determine only from a graph, was "the number in front of *x*." All the time that they had been finding a pattern in the table, they were actually finding the slope, or constant rate of change. And every time that they had written the function rule, the slope was what they multiplied their *x* value by, making it the coefficient in their equation. Mr. Conrey could see students, one by one, having their private "aha" moments as the connection between what they previously knew about tables and function rules fused with what they had recently learned about slope.

The connections did not stop there. Mr. Conrey then challenged his students to find how the constant in their equation was related to their table and graph. Students worked together in teams, trying to determine a link. Once each team had a conjecture, that team began sharing its ideas with the entire class. The connections between the constant, the ordered pair (0, __), and the *y*-intercept on the graph began popping up around the room. Students saw the connection between how they had previously determined a function rule by multiplying the slope by the *x*-value of 0 and adding or subtracting the constant to determine the *y*-value. They were able to relate an equation in slope-intercept form to their prior knowledge of function rules and tables.

Mr. Conrey could have taught slope-intercept form by directly telling his students that the coefficient is the slope and the constant is the *y*-intercept. Instead, he chose to connect the new information to students' prior knowledge of tables, function rules, and slope. The students were thus able to build on their former mathematical understanding rather than learning the concept of slope-intercept form as an isolated topic. The teacher not only established the connection between prior knowledge and new learning but also helped students make mathematical connections between multiple representations of tables, equations, and graphs.

DIRECTING ATTENTION THROUGH ENGAGEMENT

Directing students' attention throughout the instructional process is a third key component of effective instruction. Common sense tells us that for students to learn the material, they need to be focused and engaged. Effective teachers use the learning environment and instructional materials to direct students' attention, engage them in the concepts, and ultimately increase their knowledge.

Effective teaching engages students in well-chosen tasks that pique their curiosity and draw them into the mathematics (NCTM 2000), and students who engage in worthwhile mathematical tasks and whose attention is continually directed toward mathematical understanding are more likely to succeed.

The following classroom example shows how a teacher engages students in a new concept.

Mrs. Quency wanted her students to truly understand exponential functions, so she introduced the concept by posing a problem with a motivating topic: money. She grouped her students into teams of three or four and asked each team to choose a payment option. Option 1 offered a flat 1 million dollars for the team members to split among themselves. Option 2 offered the team all the pennies on the 64th square of a checkerboard when one penny is placed on the first square, two pennies on the second square, four pennies on the third square, eight pennies on the fourth square, and so on, doubling the number of pennies for each subsequent square on the checkerboard. To get the students started, Mrs. Quency gave each team a checkerboard and a bag full of pennies.

Engaged in the goal of choosing the largest amount of money, students quickly began working to determine how much money they could get with option 2. Most teams spent several minutes determining the number of pennies on each square. At about the third row of the checkerboard, students started realizing that it was going to take them a long time to get to the 64th square, and they started looking for a shortcut or a pattern.

Team discussions about repeatedly multiplying by 2 led to the idea of exponents. Students were amazed at how large the number grew and were soon convinced that option 2 was the best deal. After determining that there would be 2^{63} pennies on the last square, teams discussed how much money that was and how Mrs. Quency was going to transport all those pennies; to help with these calculations, they investigated a website (http://www.kokogiak.com/megapenny/) that visually represents 1 million, 1 billion, and up to 1 quintillion stacked pennies.

As Mrs. Quency discussed the findings with the class, students were intrigued by how and why the

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number of pennies grew so rapidly. The concept of exponential growth meant more to them than simple computation, and the money problem set up future work with exponents. Students had begun to make sense of exponential functions.

Mrs. Quency engaged all her students by giving them a problem that interested them. The teams helped direct students' attention, and students stayed engaged with the mathematics because they had other students with whom to discuss their ideas. Adding manipulatives and technology to the lesson helped get more students involved in gaining a deeper understanding of exponential functions.

EFFECTIVE INSTRUCTION LEADS TO STUDENT ACHIEVEMENT

Teaching and learning are complex processes, and teachers are the central influence. Teachers affect student achievement in many ways—through instruction, by using what they learn in collaboration with colleagues, through continual self-reflection, by building relationships with students, and by increasing their own knowledge of mathematics. Although a precise prescription for effective instruction does not yet exist, some key components of effective instruction include developing conceptual understanding, making curricular connections, and engaging students while efficiently directing their attention. Conceptual understanding and mathematical connections are strategically placed in a lesson, whereas directing student attention needs to occur throughout the lesson. Each instructional feature discussed here is vital to implementing effective instruction.

Teachers can directly apply the three features of effective instruction to their daily lesson plans and unit plans. When planning a lesson, teachers may benefit from thinking about how to teach the mathematics conceptually, make connections, and engage students. Then, after teaching a lesson, teachers who reflect on their instruction with respect to these three elements can specifically determine what parts of the lesson were and were not effective and make changes for future lessons.

By focusing on these three key components, teachers, mathematics coaches, and administrators can share a common definition of effective instruction and can work together toward specific goals. Coaches, administrators, and teacher educators can be more methodical, precise, and helpful when supporting teachers if key elements are closely defined and examined. Effective instruction can translate into higher student achievement. By attending to these three important features, educators are choosing to improve instruction and student achievement.

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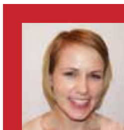

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